

# Investigating the Impact of Adaptation Sampling in Natural Evolution Strategies on Black-box Optimization Testbeds

Tom Schaul

Courant Institute of Mathematical Sciences, New York University  
Broadway 715, New York, USA  
schaul@cims.nyu.edu

## ABSTRACT

Natural Evolution Strategies (NES) are a recent member of the class of real-valued optimization algorithms that are based on adapting search distributions. Exponential NES (xNES) are the most common instantiation of NES, and particularly appropriate for the BBOB 2012 benchmarks, given that many are non-separable, and their relatively small problem dimensions. The technique of *adaptation sampling*, which adapts learning rates online further improves the algorithm’s performance. This report provides an extensive empirical comparison to study the impact of adaptation sampling in xNES, both on the noise-free and noisy BBOB testbeds.

## Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Evolution Strategies, Natural Gradient, Benchmarking

## 1. INTRODUCTION

Evolution strategies (ES), in contrast to traditional evolutionary algorithms, aim at repeating the type of mutation that led to those good individuals. We can characterize those mutations by an explicitly parameterized *search distribution* from which new candidate samples are drawn, akin to estimation of distribution algorithms (EDA). Covariance matrix adaptation ES (CMA-ES [10]) innovated the field by introducing a parameterization that includes the full covariance matrix, allowing them to solve highly non-separable problems.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

GECCO’12, July 7–11, 2012, Philadelphia, USA.

Copyright 2012 ACM 978-1-4503-0073-5/10/07 ...\$10.00.

A more recent variant, *natural evolution strategies* (NES [20, 6, 18, 19]) aims at a higher level of generality, providing a procedure to update the search distribution’s parameters for any type of distribution, by ascending the gradient towards higher expected fitness. Further, it has been shown [15, 12] that following the *natural gradient* to adapt the search distribution is highly beneficial, because it appropriately normalizes the update step with respect to its uncertainty and makes the algorithm scale-invariant.

Exponential NES (xNES), the most common instantiation of NES, used a search distribution parameterized by a mean vector and a full covariance matrix, and is thus most similar to CMA-ES (in fact, the precise relation is described in [4] and [5]). Given the relatively small problem dimensions of the BBOB benchmarks, and the fact that many are non-separable, it is also among the most appropriate NES variants for the task.

*Adaptation sampling* (AS) is a technique for the online adaptation of its learning rate, which is designed to speed up convergence. This may be beneficial to algorithms like xNES, because the optimization traverses qualitatively different phases, during which different learning rates may be optimal.

In this report, we retain the original formulation of xNES (including all parameter settings, except for an added stopping criterion), and compare this base-line algorithm with an AS-augmented variant. We describe the empirical performance on all 54 benchmark functions (both noise-free and noisy) of the BBOB 2012 workshop.

## 2. NATURAL EVOLUTION STRATEGIES

Natural evolution strategies (NES) maintain a search distribution  $\pi$  and adapt the distribution parameters  $\theta$  by following the *natural gradient* [1] of expected fitness  $J$ , that is, maximizing

$$J(\theta) = \mathbb{E}_\theta[f(\mathbf{z})] = \int f(\mathbf{z}) \pi(\mathbf{z} | \theta) d\mathbf{z}$$

Just like their close relative CMA-ES [10], NES algorithms are invariant under monotone transformations of the fitness function and linear transformations of the search space. Each iteration the algorithm produces  $n$  samples  $\mathbf{z}_i \sim \pi(\mathbf{z} | \theta)$ ,  $i \in \{1, \dots, n\}$ , i.i.d. from its search distribution, which is parameterized by  $\theta$ . The gradient w.r.t. the parameters  $\theta$  can be rewritten (see [20]) as

$$\nabla_\theta J(\theta) = \nabla_\theta \int f(\mathbf{z}) \pi(\mathbf{z} | \theta) d\mathbf{z} = \mathbb{E}_\theta [f(\mathbf{z}) \nabla_\theta \log \pi(\mathbf{z} | \theta)]$$

from which we obtain a Monte Carlo estimate

$$\nabla_{\theta} J(\theta) \approx \frac{1}{n} \sum_{i=1}^n f(\mathbf{z}_i) \nabla_{\theta} \log \pi(\mathbf{z}_i | \theta)$$

of the search gradient. The key step then consists in replacing this gradient by the natural gradient defined as  $\mathbf{F}^{-1} \nabla_{\theta} J(\theta)$  where  $\mathbf{F} = \mathbb{E} \left[ \nabla_{\theta} \log \pi(\mathbf{z} | \theta) \nabla_{\theta} \log \pi(\mathbf{z} | \theta)^{\top} \right]$  is the Fisher information matrix. The search distribution is iteratively updated using natural gradient ascent

$$\theta \leftarrow \theta + \eta \mathbf{F}^{-1} \nabla_{\theta} J(\theta)$$

with learning rate parameter  $\eta$ .

## 2.1 Exponential NES

While the NES formulation is applicable to arbitrary parameterizable search distributions [20, 12], the most common variant employs multinormal search distributions. For that case, two helpful techniques were introduced in [6], namely an exponential parameterization of the covariance matrix, which guarantees positive-definiteness, and a novel method for changing the coordinate system into a “natural” one, which makes the algorithm computationally efficient. The resulting algorithm, NES with a multivariate Gaussian search distribution and using both these techniques is called *xNES*, and the pseudocode is given in Algorithm 1.

---

### Algorithm 1: Exponential NES (xNES)

---

**input:**  $f, \boldsymbol{\mu}_{\text{init}}, \eta_{\sigma}, \eta_{\mathbf{B}}, u_k$

initialize  $\boldsymbol{\mu} \leftarrow \boldsymbol{\mu}_{\text{init}}$   
 $\sigma \leftarrow 1$   
 $\mathbf{B} \leftarrow \mathbb{I}$

**repeat**

**for**  $k = 1 \dots n$  **do**

    draw sample  $\mathbf{s}_k \sim \mathcal{N}(0, \mathbb{I})$   
     $\mathbf{z}_k \leftarrow \boldsymbol{\mu} + \sigma \mathbf{B}^{\top} \mathbf{s}_k$   
    evaluate the fitness  $f(\mathbf{z}_k)$

**end**

  sort  $\{(\mathbf{s}_k, \mathbf{z}_k)\}$  with respect to  $f(\mathbf{z}_k)$   
  and assign utilities  $u_k$  to each sample

  compute gradients

$$\begin{aligned} \nabla_{\delta} J &\leftarrow \sum_{k=1}^n u_k \cdot \mathbf{s}_k \\ \nabla_{\mathbf{M}} J &\leftarrow \sum_{k=1}^n u_k \cdot (\mathbf{s}_k \mathbf{s}_k^{\top} - \mathbb{I}) \\ \nabla_{\sigma} J &\leftarrow \text{tr}(\nabla_{\mathbf{M}} J) / d \\ \nabla_{\mathbf{B}} J &\leftarrow \nabla_{\mathbf{M}} J - \nabla_{\sigma} J \cdot \mathbb{I} \end{aligned}$$

  update parameters

$$\begin{aligned} \boldsymbol{\mu} &\leftarrow \boldsymbol{\mu} + \sigma \mathbf{B} \cdot \nabla_{\delta} J \\ \sigma &\leftarrow \sigma \cdot \exp(\eta_{\sigma} / 2 \cdot \nabla_{\sigma} J) \\ \mathbf{B} &\leftarrow \mathbf{B} \cdot \exp(\eta_{\mathbf{B}} / 2 \cdot \nabla_{\mathbf{B}} J) \end{aligned}$$

**until** *stopping criterion is met*

---

## 2.2 Adaptation Sampling

First introduced in [12] (chapter 2, section 4.4), *adaptation sampling* is a new meta-learning technique [17] that can adapt hyper-parameters online, in an economical way that is grounded on a measure statistical improvement.

Here, we apply it to the learning rate of the global step-size  $\eta_{\sigma}$ . The idea is to consider whether a larger learning-

rate  $\eta'_{\sigma} = \frac{3}{2} \eta_{\sigma}$  would have been more likely to generate the good samples in the current batch. For this we determine the (hypothetical) search distribution that would have resulted from such a larger update  $\pi(\cdot | \theta')$ . Then we compute importance weights

$$w'_k = \frac{\pi(\mathbf{z}_k | \theta')}{\pi(\mathbf{z}_k | \theta)}$$

for each of the  $n$  samples  $\mathbf{z}_k$  in our current population, generated from the actual search distribution  $\pi(\cdot | \theta)$ . We then conduct a *weighted* Mann-Whitney test [12] (appendix A) to determine if the set  $\{\text{rank}(\mathbf{z}_k)\}$  is inferior to its reweighted counterpart  $\{w'_k \cdot \text{rank}(\mathbf{z}_k)\}$  (corresponding to the larger learning rate), with statistical significance  $\rho$ . If so, we increase the learning rate by a factor of  $1 + c'$ , up to at most  $\eta_{\sigma} = 1$  (where  $c' = 0.1$ ). Otherwise it decays to its initial value:

$$\eta_{\sigma} \leftarrow (1 - c') \cdot \eta_{\sigma} + c' \cdot \eta_{\sigma, \text{init}}$$

The procedure is summarized in algorithm 2 (for details and derivations, see [12]). The combination of xNES with adaptation sampling is dubbed *xNES-as*.

One interpretation of why adaptation sampling is helpful is that half-way into the search, (after a local attractor has been found, e.g., towards the end of the valley on the Rosenbrock benchmarks  $f_8$  or  $f_9$ ), the convergence speed can be boosted by an increased learning rate. For such situations, an online adaptation of hyper-parameters is inherently well-suited.

---

### Algorithm 2: Adaptation sampling

---

**input** :  $\eta_{\sigma, t}, \eta_{\sigma, \text{init}}, \theta_t, \theta_{t-1}, \{(\mathbf{z}_k, f(\mathbf{z}_k))\}, c', \rho$

**output:**  $\eta_{\sigma, t+1}$

compute hypothetical  $\theta'$ , given  $\theta_{t-1}$  and using  $3/2 \eta_{\sigma, t}$

**for**  $k = 1 \dots n$  **do**

$$w'_k = \frac{\pi(\mathbf{z}_k | \theta')}{\pi(\mathbf{z}_k | \theta)}$$

**end**

$S \leftarrow \{\text{rank}(\mathbf{z}_k)\}$

$S' \leftarrow \{w'_k \cdot \text{rank}(\mathbf{z}_k)\}$

**if** *weighted-Mann-Whitney*( $S, S'$ )  $< \rho$  **then**

**return**  $(1 - c') \cdot \eta_{\sigma} + c' \cdot \eta_{\sigma, \text{init}}$

**else**

**return**  $\min((1 + c') \cdot \eta_{\sigma}, 1)$

**end**

---

## 3. EXPERIMENTAL SETTINGS

We use identical default hyper-parameter values for all benchmarks (both noisy and noise-free functions), which are taken from [6, 12]. Table 1 summarizes all the hyper-parameters used.

In addition, we make use of the provided target fitness  $f_{\text{opt}}$  to trigger *independent* algorithm restarts<sup>1</sup>, using a simple ad-hoc procedure: If the log-progress during the past  $1000d$

<sup>1</sup>It turns out that this use of  $f_{\text{opt}}$  is technically not permitted by the BBOB guidelines, so strictly speaking a different restart strategy should be employed, for example the one described in [12].

**Table 1: Default parameter values for xNES (including the utility function and adaptation sampling) as a function of problem dimension  $d$ .**

parameter	default value
$n$	$4 + \lceil 3 \log(d) \rceil$
$\eta_\sigma = \eta_{\mathbf{B}}$	$\frac{3(3 + \log(d))}{5d\sqrt{d}}$
$u_k$	$\frac{\max(0, \log(\frac{n}{2} + 1) - \log(k))}{\sum_{j=1}^n \max(0, \log(\frac{n}{2} + 1) - \log(j))} - \frac{1}{n}$
$\rho$	$\frac{1}{2} - \frac{1}{3(d+1)}$
$c'$	$\frac{1}{10}$

evaluations is too small, i.e., if

$$\log_{10} \left| \frac{f_{\text{opt}} - f_t}{f_{\text{opt}} - f_{t-1000d}} \right| < (r+2)^2 \cdot m^{3/2} \cdot [\log_{10} |f_{\text{opt}} - f_t| + 8]$$

where  $m$  is the remaining budget of evaluations divided by  $1000d$ ,  $f_t$  is the best fitness encountered until evaluation  $t$  and  $r$  is the number of restarts so far. The total budget is  $10^5 d^{3/2}$  evaluations.

Implementations of this and other NES algorithm variants are available in Python through the PyBrain machine learning library [16], as well as in other languages at [www.idisia.ch/~tom/nas.html](http://www.idisia.ch/~tom/nas.html).

## 4. RESULTS

Results from experiments according to [7] on the benchmark functions given in [2, 8, 3, 9] are presented in Figures 1, 2 and 3, and in Tables 2, 3 and 4. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [7, 11]. **Statistical significance** is tested with the rank-sum test for a given target  $\Delta f_t$  ( $10^{-8}$  using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by  $-1$ ), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Some of the result plots (like performance scaling with dimension), as well as the CPU-timing results were omitted here but are available in stand-alone benchmarking reports [14, 13].

## 5. DISCUSSION

Naturally, the core algorithm (xNES) being identical in both variants presented here, we do not expect drastically different results. In fact, the direct comparison Figures 1 and 2 show remarkably similar results on the majority of functions, which we see from the points clearly scattered along the diagonal. This indicates that on most benchmarks,

adaptation sampling is not used (much) and xNES-as operates with the same conservative learning rates as xNES.

The functions where we observe differences are telling, however. Most notably on the sphere function ( $f_1$ , and its noisy siblings), adaptation sampling drastically improves performance, by up to a factor 4. Adaptation sampling still improves performance significantly on many other functions (see Table 2), but the speed-ups are less striking. There is a single function where adaptation sampling significantly hurts performance, namely on  $f_{13}$  in dimension 20 (the sharp ridge): apparently the learning rates are increased to an overly aggressive level.

In conclusion, we can recommend using adaptation sampling for xNES in general, and particularly so if the fitness landscapes are smooth.

## Acknowledgements

The author wants to thank the organizers of the BBOB workshop for providing such a well-designed benchmark setup, and especially such high-quality post-processing utilities.

This work was funded in part through AFR postdoc grant number 2915104, of the National Research Fund Luxembourg.

## 6. REFERENCES

- [1] S. I. Amari. Natural Gradient Works Efficiently in Learning. *Neural Computation*, 10:251–276, 1998.
- [2] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.
- [3] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2010: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2010.
- [4] N. Fukushima, Y. Nagata, S. Kobayashi, and I. Ono. Proposal of distance-weighted exponential natural evolution strategies. In *2011 IEEE Congress of Evolutionary Computation*, pages 164–171. IEEE, 2011.
- [5] T. Glasmachers, T. Schaul, and J. Schmidhuber. A Natural Evolution Strategy for Multi-Objective Optimization. In *Parallel Problem Solving from Nature (PPSN)*, 2010.
- [6] T. Glasmachers, T. Schaul, Y. Sun, D. Wierstra, and J. Schmidhuber. Exponential Natural Evolution Strategies. In *Genetic and Evolutionary Computation Conference (GECCO)*, Portland, OR, 2010.
- [7] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.
- [8] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.
- [9] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009. Updated February 2010.

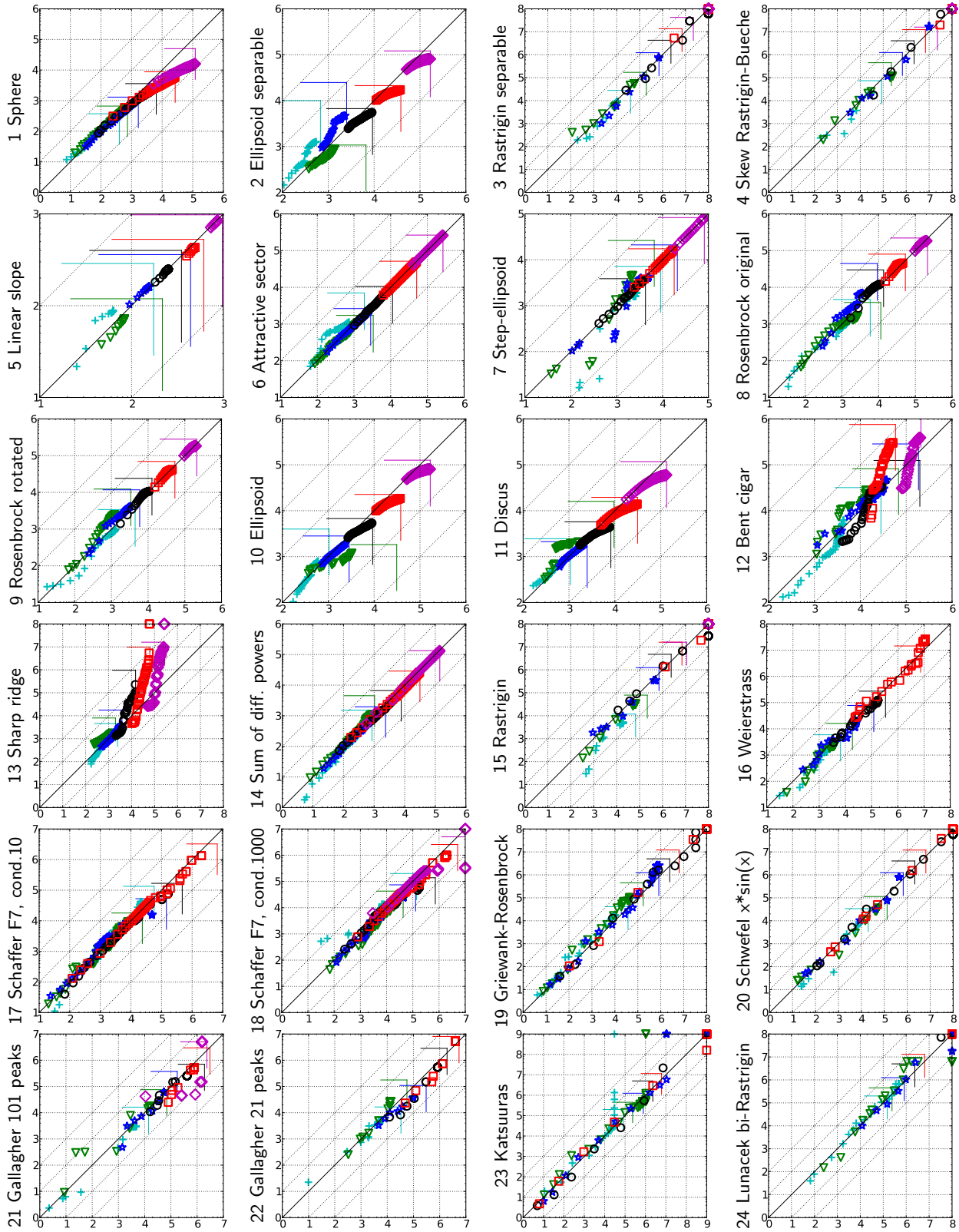


Figure 1: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of xNES ( $x$ -axis) versus xNES-as ( $y$ -axis) for 46 target values  $\Delta f \in [10^{-8}, 10]$  in each dimension on functions  $f_1$ – $f_{24}$ . Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2:+, 3:∇, 5:\*, 10:○, 20:□, 40:◇.

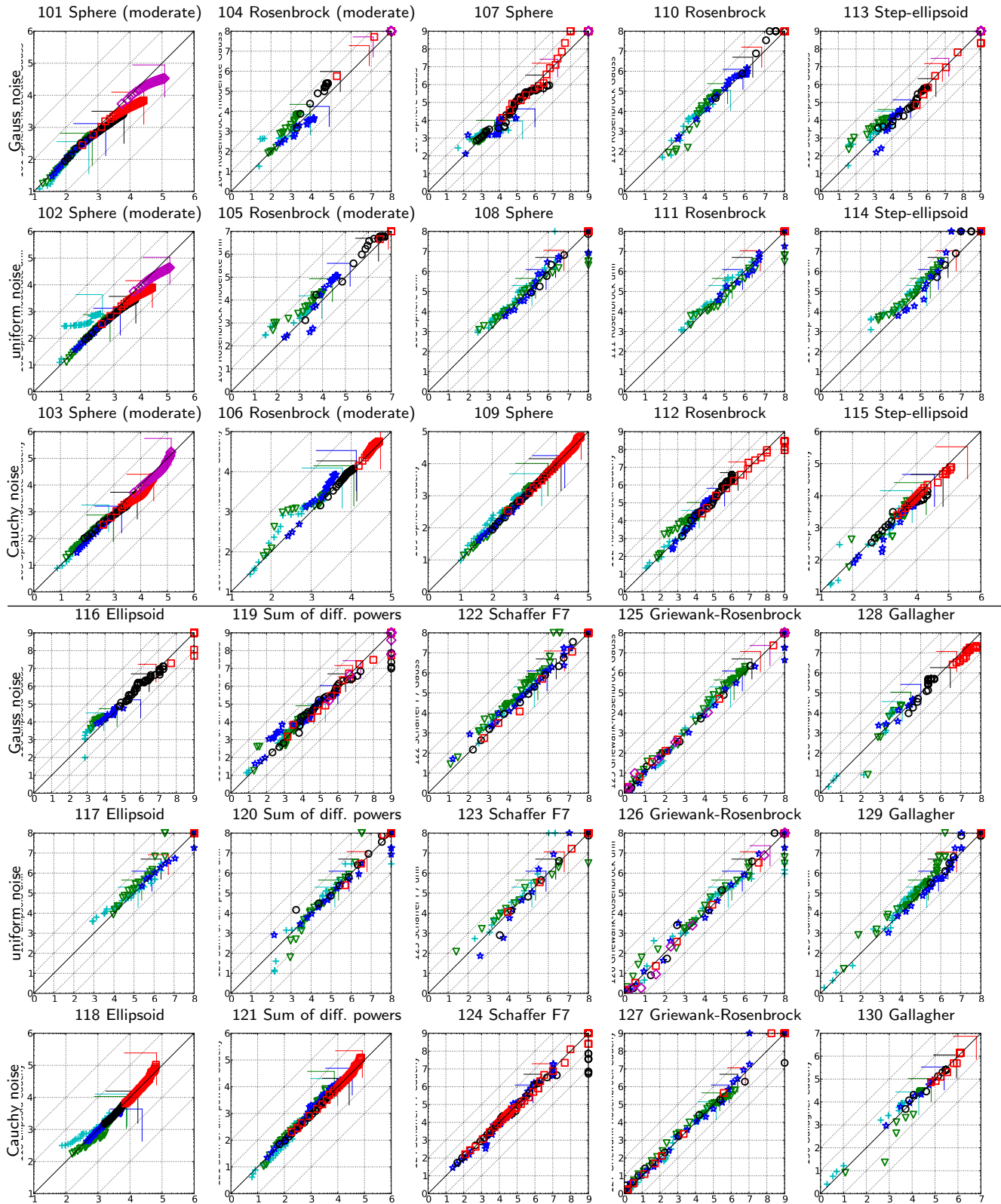


Figure 2: Expected running time (ERT in  $\log_{10}$  of number of function evaluations) of xNES ( $x$ -axis) versus xNES-as ( $y$ -axis) for 46 target values  $\Delta f \in [10^{-8}, 10]$  in each dimension on functions  $f_{101}$ – $f_{130}$ . Markers on the upper or right edge indicate that the target value was never reached. Markers represent dimension: 2: +, 3:  $\nabla$ , 5: \*, 10: o, 20:  $\square$ , 40:  $\diamond$ .





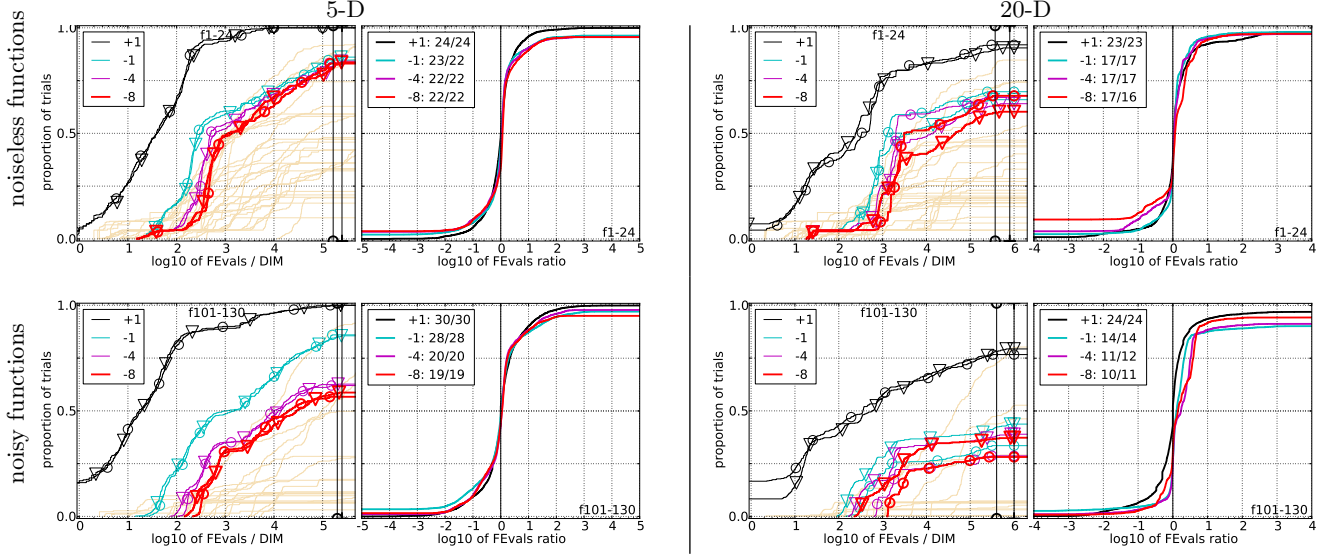


Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension  $D$  (FEvals/ $D$ ) to reach a target value  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k \in \{1, -1, -4, -8\}$  is given by the first value in the legend, for xNES ( $\circ$ ) and xNES-as ( $\nabla$ ). Light beige lines show the ECDF of FEvals for target value  $\Delta f = 10^{-8}$  of all algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of xNES divided by xNES-as, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being  $> 0$  or  $< 1$ . The legends indicate the number of functions that were solved in at least one trial (xNES first).

		5-D					20-D							
$\Delta f$		1e+1	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f$	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
$f_{101}$		11	44	62	69	75	15/15	$f_{101}$	59	571	700	739	783	15/15
1: xNES		3.4(1)	5.4(0.9)	10(1)	14(1)	18(0.6)	15/15	1: xNES	5.5(2)	10(0.7)	17(0.6)	24(0.4)	30(0.6)	15/15
2: +as		2.8(2)	5.2(2)	7.9(2)*	10(2)*3	13(3)*4	15/15	2: +as	4.8(2)	5.7(2)*5	6.7(2)*5	7.7(3)*5	8.4(3)*5	30/30
$f_{102}$		11	50	72	86	99	15/15	$f_{102}$	231	579	921	1157	1407	15/15
1: xNES		3.1(4)	4.9(1)	8.8(1)	12(1)	14(1)	15/15	1: xNES	1.5(0.7)	10(0.5)	13(0.3)	15(0.2)	17(0.4)	15/15
2: +as		3.3(3)	4.7(1)	7.3(1)*2	9.2(1)*3	10(2)*4	15/15	2: +as	1.5(0.4)	6.2(2)*5	5.6(2)*5	5.4(2)*5	5.4(2)*5	30/30
$f_{103}$		11	30	31	35	115	15/15	$f_{103}$	65	629	1313	1893	2464	14/15
1: xNES		3.7(2)	8.2(2)	20(3)	31(3)	13(1)	15/15	1: xNES	6.2(2)	10(0.4)	9.4(0.3)	10(0.4)	11(0.3)	15/15
2: +as		2.7(2)	7.2(2)	16(3)*2	24(5)*2	11(2)*2	15/15	2: +as	5.0(2)	4.9(2)*5	4.1(2)*5	5.5(2)*5	6.8(2)*5	30/30
$f_{104}$		173	1287	1768	2040	2284	15/15	$f_{104}$	23690	1.7e5	1.8e5	1.9e5	2.0e5	15/15
1: xNES		1.4(0.7)	5.6(10)	7.3(11)	6.4(10)	5.9(9)	15/15	1: xNES	8.1(10)	$\infty^*$	$\infty^*$	$\infty^*$	$\infty^{8.0e6}$	0/15
2: +as		1.4(0.4)	2.7(6)	2.3(4)	2.1(4)	2.0(3)	15/15	2: +as	24(29)	$\infty$	$\infty$	$\infty$	$\infty^{1.8e7}$	0/30
$f_{105}$		167	5174	10388	10824	11202	15/15	$f_{105}$	1.9e5	6.3e5	6.5e5	6.6e5	6.7e5	15/15
1: xNES		1.3(0.5)	4.9(7)	3.6(5)	4.0(5)	4.0(5)	15/15	1: xNES	17(16)	$\infty$	$\infty$	$\infty$	$\infty^{7.4e6}$	0/15
2: +as		1.4(0.5)	14(13)	11(7)	11(7)	10(7)	15/15	2: +as	24(22)	$\infty$	$\infty$	$\infty$	$\infty^{1.6e7}$	0/21
$f_{106}$		92	1050	2666	2887	3087	15/15	$f_{106}$	11480	23746	25470	26492	27360	15/15
1: xNES		2.9(1.0)	2.2(0.8)	1.2(0.9)	1.2(0.9)	1.3(1)	15/15	1: xNES	1.3(0.1)	1.3(0.1)	1.3(0.1)	1.5(0.1)	1.7(0.1)	15/15
2: +as		2.7(0.8)	3.7(5)	2.2(3)	2.2(3)	2.7(3)	15/15	2: +as	1.2(0.2)*2	1.5(0.4)	1.7(0.9)	1.8(0.7)	2.0(0.7)	30/30
$f_{107}$		40	453	940	1376	1850	15/15	$f_{107}$	8571	16226	27357	52486	65052	15/15
1: xNES		3.2(3)	20(33)	10(16)	7.6(11)	45(8)	14/15	1: xNES	1.7(0.4)	879(889)	$\infty$	$\infty$	$\infty^{6.9e6}$	0/15
2: +as		3.2(5)	4.2(0.5)	6.9(11)	7.2(8)	7.0(10)	15/15	2: +as	1.6(0.9)	786(927)	$\infty$	$\infty$	$\infty^{1.5e7}$	0/15
$f_{108}$		87	14469	30935	58628	80667	15/15	$f_{108}$	58063	2.0e5	4.5e5	6.3e5	9.0e5	15/15
1: xNES		84(117)	36(27)	$\infty$	$\infty$	$\infty^{7.5e5}$	0/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty^{5.9e6}$	0/15
2: +as		69(120)	56(54)	$\infty$	$\infty$	$\infty^{1.3e6}$	0/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty^{1.1e7}$	0/15
$f_{109}$		11	216	572	873	946	15/15	$f_{109}$	333	1138	2287	3583	4952	15/15
1: xNES		3.4(2)	1.4(0.3)	2.0(0.5)	2.3(0.2)	4.1(0.7)	15/15	1: xNES	0.94(0.3)	7.7(0.4)	10(0.5)	10(0.5)	10(0.3)	15/15
2: +as		3.6(2)	1.3(0.5)	1.8(0.5)	2.3(0.6)	4.1(0.8)	15/15	2: +as	1.0(0.2)	6.9(0.8)	9.4(0.7)	10(0.5)	13(8)	15/15
$f_{110}$		949	1.2e5	5.9e5	6.0e5	6.1e5	15/15	$f_{110}$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0
1: xNES		0.49(0.2)	2.1(3)	1.7(2)	2.2(2)	2.2(2)	7/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as		0.44(0.2)	3.6(5)	1.2(1)	1.6(2)	2.3(3)	8/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$f_{111}$		6856	8.8e6	2.3e7	3.1e7	3.1e7	3/15	$f_{111}$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0
1: xNES		7.2(9)	0.61(0.7)	$\infty$	$\infty^{7.8e5}$	$\infty$	0/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as		5.4(8)	0.96(1)	$\infty$	$\infty^{1.3e6}$	$\infty$	0/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
$f_{112}$		107	3421	4502	5132	5596	15/15	$f_{112}$	25552	69621	73557	76137	78238	15/15
1: xNES		4.6(1)	3.0(3)	6.5(8)	12(15)	18(28)	15/15	1: xNES	0.93(0.2)	1454(1717)	$\infty$	$\infty$	$\infty^{7.0e6}$	0/15
2: +as		2.3(1)	7.9(10)	17(18)	28(23)	37(38)	15/15	2: +as	0.99(0.2)	952(988)	3878(4223)	3747(3922)	3646(4035)	1/15
$f_{113}$		133	8081	24128	24128	24402	15/15	$f_{113}$	50123	5.6e5	5.9e5	5.9e5	5.9e5	15/15
1: xNES		10(6)	2.1(3)	1.2(2)	1.2(2)	1.2(2)	15/15	1: xNES	5.0(5)	$\infty$	$\infty$	$\infty$	$\infty^{6.8e6}$	0/15
2: +as		1.2(1)	2.5(4)	1.4(2)	1.4(2)	1.5(2)	15/15	2: +as	1.4(1)	$\infty$	$\infty$	$\infty$	$\infty^{1.5e7}$	0/15
$f_{114}$		767	56311	83272	83272	84949	15/15	$f_{114}$	2.1e5	1.4e6	1.6e6	1.6e6	1.6e6	15/15
1: xNES		10(17)	33(39)	$\infty$	$\infty$	$\infty^{7.5e5}$	0/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty^{5.8e6}$	0/15
2: +as		8.0(13)	157(161)	$\infty$	$\infty$	$\infty^{1.3e6}$	0/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty^{1.0e7}$	0/15
$f_{115}$		64	1829	2550	2550	2970	15/15	$f_{115}$	2405	91749	1.3e5	1.3e5	1.3e5	15/15
1: xNES		1.7(1)	2.3(5)	2.5(5)	2.5(5)	2.6(4)	15/15	1: xNES	1.0(0.2)	0.14(0.0) <sup>↓</sup>	0.62(0.8)	0.62(0.8)	0.69(0.8)	15/15
2: +as		1.3(0.9)	1.8(4)	1.8(4)	1.8(4)	1.6(3)	15/15	2: +as	1.0(0.1)	0.14(0.0) <sup>↓</sup>	0.37(0.3) <sup>↓3</sup>	0.37(0.3) <sup>↓3</sup>	0.46(0.5) <sup>↓2</sup>	15/15

Table 3: Relative ERT in number of  $f$ -evaluations, see Table 2 for details.

$\Delta f$	1e+1	1e-1	1e-3	1e-5	1e-7	#succ	$\Delta f$	1e+1	1e-1	1e-3	1e-5	1e-7	#succ
<b>f<sub>116</sub></b>	5730	22311	26868	30329	31661	15/15	<b>f<sub>116</sub></b>	5.0e5	8.9e5	1.0e6	1.1e6	1.1e6	15/15
1: xNES	0.62(1)	1.3(2)	1.2(1)	1.2(1)	1.2(1)	14/15	1: xNES	101(110)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as	1.4(2)	1.1(2)	1.3(1)	1.6(2)	1.8(2)	15/15	2: +as	39(42)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>117</sub></b>	26686	1.1e5	1.4e5	1.7e5	1.9e5	15/15	<b>f<sub>117</sub></b>	1.8e6	2.6e6	2.9e6	3.2e6	3.6e6	15/15
1: xNES	9.4(13)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as	8.4(6)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/30
<b>f<sub>118</sub></b>	429	1555	1998	2430	2913	15/15	<b>f<sub>118</sub></b>	6908	17514	26342	30062	32659	15/15
1: xNES	1.0(0.4)	0.52(0.1) <sup>14</sup>	0.73(0.1) <sup>12</sup>	1.0(0.2)	1.6(0.2)	15/15	1: xNES	1.0(0.0)	0.90(0.0)	1.1(0.1)	1.5(0.1)	1.8(0.0)	15/15
2: +as	1.00(0.4)	0.48(0.1) <sup>14</sup>	0.69(0.2) <sup>12</sup>	0.96(0.2)	1.1(0.2)	15/15	2: +as	<b>0.94(0.1)*3</b>	<b>0.80(0.1)*3</b>	1.1(0.1)	1.7(0.7)	2.1(0.7)	15/15
<b>f<sub>119</sub></b>	12	1136	10372	35296	49747	15/15	<b>f<sub>119</sub></b>	2771	35930	4.1e5	1.4e6	1.9e6	15/15
1: xNES	1.9(2)	0.66(0.2)	2.6(3)	1.5(1)	12(13)	9/15	1: xNES	0.54(0.3) <sup>12</sup>	2741(2972)	$\infty$	$\infty$	$\infty$	0/15
2: +as	3.8(4)	6.3(13)	1.6(2)	2.3(3)	5.9(6)	15/15	2: +as	0.53(0.4) <sup>1</sup>	851(771)	$\infty$	$\infty$	$\infty$	0/13
<b>f<sub>120</sub></b>	16	18698	72438	3.3e5	5.5e5	15/15	<b>f<sub>120</sub></b>	36040	2.8e5	1.6e6	6.7e6	1.4e7	13/15
1: xNES	9.0(12)	43(39)	$\infty$	$\infty$	$\infty$	0/15	1: xNES	14(23)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as	51(18)	25(33)	$\infty$	$\infty$	$\infty$	0/15	2: +as	6.9(7)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>121</sub></b>	8.6	273	1583	3870	6195	15/15	<b>f<sub>121</sub></b>	249	1426	9304	34434	57404	15/15
1: xNES	2.7(3)	1.2(0.2)	1.3(0.5)	1.2(1)	1.7(2)	15/15	1: xNES	0.74(0.2)	6.7(0.4)	2.9(0.2)	1.3(0.1)	1.1(0.1)	15/15
2: +as	2.6(2)	0.96(0.5)	1.6(0.8)	1.5(2)	2.3(3)	15/15	2: +as	0.81(0.3)	6.3(0.5)	2.9(0.2)	1.7(0.6)	1.7(1)	15/15
<b>f<sub>122</sub></b>	10	9190	30087	53743	1.1e5	15/15	<b>f<sub>122</sub></b>	692	1.4e5	7.9e5	2.0e6	5.8e6	15/15
1: xNES	1.7(2)	6.4(4)	200(215)	$\infty$	$\infty$	0/15	1: xNES	0.91(0.8)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as	5.0(4)	8.9(8)	583(686)	$\infty$	$\infty$	0/15	2: +as	0.81(0.9)	$\infty$	$\infty$	$\infty$	$\infty$	0/13
<b>f<sub>123</sub></b>	11	81505	3.4e5	6.7e5	2.2e6	15/15	<b>f<sub>123</sub></b>	1063	1.5e6	5.3e6	2.7e7	1.6e8	0
1: xNES	36(70)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	1: xNES	9.0(9)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
2: +as	6.6(9)	$\infty$	$\infty$	$\infty$	$\infty$	0/15	2: +as	11(14)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>124</sub></b>	10	1040	20478	45337	95200	15/15	<b>f<sub>124</sub></b>	192	40840	1.3e5	3.9e5	8.0e5	15/15
1: xNES	2.5(2)	3.8(10)	2.0(2)	19(20)	112(127)	1/15	1: xNES	0.63(0.3)	0.58(0.1)	6.4(7)	$\infty$	$\infty$	0/15
2: +as	2.9(3)	2.0(0.7)	1.1(1)	36(40)	60(66)	1/15	2: +as	0.84(0.3)	0.59(0.1)	4.0(3)	$\infty$	$\infty$	0/15
<b>f<sub>125</sub></b>	1	1	2.4e5	2.4e5	2.5e5	15/15	<b>f<sub>125</sub></b>	1	1	2.5e7	8.0e7	8.1e7	4/15
1: xNES	1.1	9476(14165)	$\infty$	$\infty$	$\infty$	0/15	1: xNES	1.5(1)	$\infty$	$\infty$	$\infty$	$\infty$	0/30
2: +as	1.3(0.5)	7786(7916)	$\infty$	$\infty$	$\infty$	0/15	2: +as	1.7(1)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>126</sub></b>	1	1	$\infty$	$\infty$	$\infty$	0	<b>f<sub>126</sub></b>	1	1	$\infty$	$\infty$	$\infty$	0
1: xNES	1.1	44499(50671)	$\infty$	$\infty$	$\infty$	0/15	1: xNES	1.2(0.5)	$\infty$	$\infty$	$\infty$	$\infty$	0/30
2: +as	1.1(0.5)	41598(46392)	$\infty$	$\infty$	$\infty$	0/15	2: +as	1.4(1)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>127</sub></b>	1	1	3.4e5	3.9e5	4.0e5	15/15	<b>f<sub>127</sub></b>	1	1	4.4e6	7.3e6	7.4e6	15/15
1: xNES	1.1	3395(5420)	31(36)	$\infty$	$\infty$	0/15	1: xNES	1.4(1)	$\infty$	$\infty$	$\infty$	$\infty$	0/30
2: +as	1.1(0.5)	3858(5345)	$\infty$	$\infty$	$\infty$	0/15	2: +as	1.5(0.5)	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>128</sub></b>	111	7808	12447	17217	21162	15/15	<b>f<sub>128</sub></b>	1.4e5	1.7e7	1.7e7	1.7e7	1.7e7	9/15
1: xNES	7.1(2)	5.2(6)	3.3(4)	2.4(3)	2.0(2)	15/15	1: xNES	28(32)	1.2(1)	2.0(2)	3.4(4)	3.4(4)	3/30
2: +as	22(67)	5.7(7)	3.6(4)	2.6(3)	2.1(2)	15/15	2: +as	19(23)	0.78(0.9)	1.0(1)	1.0(1.0)	1.3(1)	6/15
<b>f<sub>129</sub></b>	64	59443	2.8e5	5.1e5	5.8e5	15/15	<b>f<sub>129</sub></b>	7.8e6	4.2e7	4.2e7	4.2e7	4.2e7	5/15
1: xNES	38(34)	10(8)	11(13)	$\infty$	$\infty$	0/15	1: xNES	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/30
2: +as	17(16)	8.6(12)	14(16)	$\infty$	$\infty$	0/15	2: +as	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0/15
<b>f<sub>130</sub></b>	55	3034	32823	33889	34528	10/15	<b>f<sub>130</sub></b>	4904	2.5e5	2.5e5	2.6e5	2.6e5	7/15
1: xNES	13(1)	25(25)	2.4(2)	2.3(2)	2.3(2)	15/15	1: xNES	14(20)	4.9(6)	4.9(5)	4.9(5)	4.9(5)	29/30
2: +as	17(0.9)	28(48)	2.6(4)	2.5(4)	2.5(4)	15/15	2: +as	15(23)	5.4(9)	5.4(9)	5.4(9)	5.4(9)	15/15

Table 4: Relative ERT in number of  $f$ -evaluations, see Table 2 for details.

- [10] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *IEEE Transactions on Evolutionary Computation*, 9:159–195, 2001.
- [11] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.
- [12] T. Schaul. *Studies in Continuous Black-box Optimization*. Ph.D. thesis, Technische Universität München, 2011.
- [13] T. Schaul. Benchmarking Exponential Natural Evolution Strategies on the Noiseless and Noisy Black-box Optimization Testbeds. In *Black-box Optimization Benchmarking Workshop, Genetic and Evolutionary Computation Conference*, Philadelphia, PA, 2012.
- [14] T. Schaul. Benchmarking Natural Evolution Strategies with Adaptation Sampling on the Noiseless and Noisy Black-box Optimization Testbeds. In *Black-box Optimization Benchmarking Workshop, Genetic and Evolutionary Computation Conference*, Philadelphia, PA, 2012.
- [15] T. Schaul. Natural Evolution Strategies Converge on Sphere Functions. In *Genetic and Evolutionary Computation Conference (GECCO)*, Philadelphia, PA, 2012.
- [16] T. Schaul, J. Bayer, D. Wierstra, Y. Sun, M. Felder, F. Sehnke, T. Rückstieß, and J. Schmidhuber. PyBrain. *Journal of Machine Learning Research*, 11:743–746, 2010.
- [17] T. Schaul and J. Schmidhuber. *Metalearning. Scholarpedia*, 5(6):4650, 2010.
- [18] Y. Sun, D. Wierstra, T. Schaul, and J. Schmidhuber. Stochastic search using the natural gradient. In *International Conference on Machine Learning (ICML)*, 2009.
- [19] D. Wierstra, T. Schaul, T. Glasmachers, Y. Sun, and J. Schmidhuber. Natural Evolution Strategies. Technical report, 2011.
- [20] D. Wierstra, T. Schaul, J. Peters, and J. Schmidhuber. Natural Evolution Strategies. In *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)*, Hong Kong, China, 2008.